



Fig. 1 Coordinate system.

It should be noted that Thyson and Schurmann⁴ used a two-dimensional displacement thickness δ^* , and their formula does not fully account for the transverse curvature effect. In hypersonic boundary layers, $\delta^* \rightarrow \delta$; then, Eq. (10) becomes

$$\frac{v_e}{u_e} = \frac{\rho_w v_w r_w}{\rho_e u_e r_e} + \frac{d\delta^*}{dx} \quad (12)$$

Equation (12) provides the necessary matching condition used in Ref. 3.

References

- ¹ Li, T. Y. and Gross, J. F., "Hypersonic strong viscous interaction on a flat plate with surface mass transfer," *Proceedings of 1961 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1961), p. 146.
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Reply by Authors to T.-Y. Li and J. F. Gross

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IF transverse curvature effects are to be included, the inviscid streamline slope expression as given by Li and Gross is quite correct. In our original note, a transverse curvature effect was omitted which multiplies the rate of changes of the displacement thickness and inviscid flow. The slope is

$$\frac{V_e}{U_e} = \frac{\rho_w V_w}{\rho_e U_e} \left[\frac{1}{1 + (\delta/r)} \right] + \frac{d\delta}{dx} \left\{ 1 - \left[\frac{1}{1 + (\delta/r)} \right] \right\} + \left[\frac{d\delta^*}{dx} - \frac{\delta - \delta^*}{\rho_e U_e r} \frac{d}{dx} (\rho_e U_e r) \right] \left[\frac{1}{1 + (\delta/r)} \right]$$

The difference then between the expression as given by Li and Gross and the foregoing expression is that they have properly included a transverse curvature effect in the axisymmetry displacement thickness. However, our displacement thickness is an axisymmetric displacement thickness. What we neglected was a transverse curvature effect on the displacement thickness. Since we did not consider transverse effects, our pressure interaction analysis as such is still valid.

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Comment on "Orbital Motion in the Theory of General Relativity"

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IN their discussion¹ of relativistic orbital mechanics, Anderson and Lorell note the difficulty in interpreting the equations of motion. They go on to mention that, although their result for the secular change in the argument of pericenter agrees with that of Bogorodskii,² they do not match the latter in regard to his estimate of the effect on χ (i.e., $-\pi\tau$). The difficulty lies in the fact that, in the two papers, somewhat different perturbative components, R and S (the former radial, the latter tangential), are used. Further, Bogorodskii's error in computing changes in χ was not corrected, as is evident in Eq. (4) of Ref. 1.

That this equation is incomplete has been noted before,³ although a very careful reading of the source material⁴ is required in order to perceive this; other sources^{5,6} are only slightly less ambiguous.

We find, employing the same notation as Anderson and Lorell, and following Brouwer and Clemence⁶ (especially pages 285-286 and 300-301),

$$\frac{d\chi}{dt} = [3\mu n / ac^2(1 - e^2)] \times [2 + e^2 - (3 + 2e + e^2 + 2e^3)/(1 - e^2)]$$

Incidentally, utilizing Bogorodskii's perturbations (but setting his ω_0 equal to zero), we obtain

$$\frac{d\chi}{dt} = [3\mu n / c^2 a(1 - e^2)^{1/2}] \times [2 - 2e^2 - (5 + 7e + 2e^2 + 3e^3 + 3e^4)/(1 - e^2)^{3/2}]$$

References

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- ⁵ Smart, W. M., *Celestial Mechanics* (Longmans Green and Co., Inc., London, 1953), pp. 82-86.
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Reply by Author to H. R. Westerman

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THE comment by H. R. Westerman is exemplary of a certain amount of confusion that has resulted from the introduction of the mean longitude L in the relativistic perturbations of Ref. 1. The purpose of this note is to clarify the meaning of the averaged rate in the mean longitude.

When the time rate of change of the mean anomaly phase angle χ is defined as in Eq. (4) of Ref. 1, then the mean anomaly rate is given by

$$dM/dt = n + (d\chi/dt) \quad (1)$$

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